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Large magnetic dipole moments for neutrinos with arbitrary masses

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Abstract

We show that there is a general sort of models in which it is possible to have large magnetic dipole moments for neutrinos while keeping their masses arbitrarily small. Some examples of these models are considered.

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It has been known since several years ago that a way to solve the solar neutrino problem is provided by the assumption that the electron neutrino is a massive Dirac particle with a magnetic moment $\mu_\nu \leq (10^{-11} - 10^{-10})\mu_B$, where μ_B is the Bohr magneton [1,2]. The bound on the muon neutrino magnetic moment, $\mu_{\nu_\mu} < 10^{-8}\mu_B$, coming from neutral current data is larger than the one for the electron neutrino. It has been pointed out that its effect can be already observed with the magnetic field of the Earth [3,4]. However, since there is a controversy over the validity of the claimed upper limits on the magnetic moment from the astrophysical data we use for reference the values given in PDG [5]:

$$
\begin{aligned}
&\mu_{\nu_e} < 3.2 \times 10^{-10}\mu_B, \\
&\mu_{\nu_\mu} < 8.5 \times 10^{-10}\mu_B, \\
&\mu_{\nu_\tau} < 5.4 \times 10^{-7}\mu_B.
\end{aligned}
\tag{1}
$$

In the standard model its magnetic moment is rather small [6]

$$
\mu_\nu \leq (10^{-19} - 10^{-18})(m_\nu/\mathrm{eV})\mu_B. \tag{2}
$$

On the other hand, it is difficult to give to the neutrino a large magnetic moment because in most of models it is proportional to the neutrino mass. In supersymmetric theories with R-parity breaking magnetic moments of the order of $\mu_\nu \simeq 10^{-13}\mu_B$ arise [7]. This is worst in $L-R$ symmetric model in which $\mu_{\nu_e} < 10^{-4}\mu_B$ [8].

The problem is that in most models the same loop diagram without the photon line give a contribution to the neutrino mass [8,9]. To suppress the contribution to the mass some authors introduce continuous [10] or discrete symmetries [11]. It is possible to have models in which an $SU(2)_\nu$ symmetry, à la Voloshin [12], acting on (ν, ν^c) as a doublet would forbid the mass but allow the magnetic moment [13]. It is possible to implement a large μ_ν in a model with $SU(2) \otimes U(1)$ gauge symmetry but with lepton doublets $(\nu\, l^-)_L$ and $(\nu\, l^-)_R$ [14]. The problems above arise because both the neutrino mass and magnetic moment are supposed to be calculable. An economic model is a minimal version of the Zee's model [9] in which only one scalar singlet h^- (besides the usual doublet) and right-handed neutrinos are added to the $SU(2) \otimes U(1)$ model [15].

Here we will consider a general sort of models which have some of the feature of the models of Refs. [14,15] in the sense that the magnetic moment does not depend directly on the neutrino mass. A similar mechanism for an electric dipole moment for the charged leptons was proposed in Ref. [16]. In the present work we will consider a more general sort of models with or without exotic charged leptons and charged scalars.

Here we will consider that the neutrino masses are in the same foot that the masses of all fermions: they are renormalizable and for this reason, they are not calculable. Finite or infinite contributions are erased by the renormalization procedure. On the other hand the magnetic moment of all elementary fermions are both finite and calculable.

We will illustrate in this work the features of general models allowing a neutrino magnetic moment which has this characteristic: the magnetic moment is approximately independent of the neutrino mass since it appears always as a factor m_ν/v_s where v_s is a small vacuum expectation value (VEV) or it is in fact proportional only to the mass of a charged antilepton (exotic or the usual ones). Two cases are going to be considered: *i)* all neutrinos are almost degenerate in mass, so the condition $m_\nu/v_s \approx 1$ is valid for all of them; *ii)* there is a hierarchy in masses and the condition $m_\nu/v_s \approx 1$ is valid only for the heavier neutrino.

Supposing a model with right-handed neutrinos, we can parameterize the neutral and charged Higgs interactions in the lepton sector as follows:

$$\begin{aligned}-\mathcal{L}^Y &= \overline{E'_L}G^E E'_R\chi^0 + \overline{\nu'}_L G^E E'_R\chi^- \\ &+ \overline{\nu'_L}G^\nu \nu'_R\eta^0 + \overline{E'_L}G^\nu\nu'_R\eta^+ + H.c.\end{aligned} \tag{3}$$

G^E and G^ν are arbitrary complex matrices and assuming three neutrinos G^ν is a 3×3 matrix. Here E' can denote a positive charged exotic lepton E'^+ or the charge conjugated of the known charged lepton l'^+ and the primes denote symmetry eigenstates (with respect to an arbitrary electroweak symmetry). With biunitary transformations like

$$\nu'_{L,R} = \mathcal{O}^\nu_{L,R}\nu_{L,R}, \quad E'_{L,R} = \mathcal{O}^E_{L,R}E_{L,R}, \tag{4}$$

with the unprimed field denoting mass eigenstates, we can redefine the interactions in Eq. (3) in terms of the mass matrices $v_l\mathcal{O}^{E\dagger}_L G^E\mathcal{O}^E_R = M^E$, and $v_s\mathcal{O}^{\nu\dagger}_L G^\nu\mathcal{O}^\nu_R = M^\nu$, where $v_l = \langle\chi^0\rangle$ and $v_s = \langle\eta^0\rangle$ are appropriate vacuum expectation values; M^E and M^ν are real diagonal matrices, in particular $M^E = \mathrm{diag}(m_{E_1}, m_{E_2}, m_{E_3})$, $M^\nu = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$.

Next, we can rewrite the charged scalar interactions in Eq. (3) as

$$\frac{M^E}{v_l}\bar{\nu}_L K E_R\chi^- + \frac{M^\nu}{v_s}\bar{E}_L K^\dagger\nu_R S^+ + H.c., \tag{5}$$

where v_l and v_s denote a large and a small vacuum expectation value (VEV), respectively, and $K = \mathcal{O}^{\nu\dagger}_L\mathcal{O}^E_L$. Moreover, v_s is the only VEV which contributes to the neutrino masses. In some models the fraction M^E/v_l and M_ν/v_s are substituted by dimensionless Yukawa couplings (see below).

These interactions generate diagrams like the one shown in Fig. 1. Notice that one of the vertex is proportional to the neutrino mass, the other one is proportional to the mass of the charged lepton E and there is still a mass insertion of the charged lepton. Hence the magnetic moment is proportional to $m_\nu m_E^2/v_\chi v_s$ times a dimensionless function. Explicitly we have that the magnetic moment for the ν_i neutrino, arisen from diagrams in Fig. 1 and the corresponding ones with the photon line attached to the internal fermion line, is given by

$$\mu_{\nu_i} = -\frac{m_e}{4\pi^2}\frac{m_{\nu_i}}{v_s}\sum_j \mathrm{Re}\left(K^\dagger_{ij}K_{ji}\right)\frac{m_{E_j}}{v_l}\frac{m_{E_j}}{m_Y^2}\,F(m_Y, m_{E_j})\mu_B, \tag{6}$$

where $F = [F_+(m_Y, m_{E_j}) + F_-(m_Y, m_{E_j})]$ and there is no sum in i; and we have also defined

$$\begin{aligned}F_\pm(m_Y, m_{E_j}) &= -\frac{m_Y^2}{2m^2_{\nu_i}}\ln\frac{m_Y^2}{m^2_{E_j}} + \frac{m_Y^2}{2m^2_{\nu_i}\Delta} \\ &\cdot\left(m_Y^2 \pm m^2_{\nu_i} - m^2_{E_j}\right)\ln\left[\frac{m^2_{E_j} + m_Y^2 - m^2_{\nu_i} + \Delta}{m^2_{E_j} + m_Y^2 - m^2_{\nu_i} - \Delta}\right],\end{aligned} \tag{7}$$

with $\Delta^2 = [(m_Y + m_{E_i})^2 - m^2_{\nu_i}][(m_Y - m_{E_i})^2 - m^2_{\nu_i})]$, where Y^- is a mass eigenstate scalar but we have omitted the mixing among χ^- and η^- since this is a model dependent issue. Notice that in Eq. (6) we have already written μ_{ν_i} in terms of the Bohr magneton $\mu_B = e/2m_e$.

We can consider $m_{E_j}/v_l \approx 1$ and the case *i)* when the condition $m_{\nu_i}/v_s \approx 1$ is valid for all neutrinos; and in the limit $m_E, m_Y \gg m_\nu$ we can write

$$\mu_{\nu_i} \approx -\frac{m_e}{4\pi^2}\sum_j \mathrm{Re}\left(K^\dagger_{ij}K_{ji}\right) m_{E_j} \cdot \left[\frac{m_Y^2+m_{E_j}^2}{(m_Y^2-m_{E_j}^2)^2}\ln\left(\frac{m_Y^2}{m_E^2}\right) - \frac{2}{m_Y^2-m_{E_j}^2}\right]\mu_B, \tag{8}$$

With $\mathcal{K}^\dagger_{e1}\mathcal{K}_{1e} \approx 1$, $\mathcal{K}^\dagger_{e2}\mathcal{K}_{2e} \approx 10^{-5}$ and $\mathcal{K}^\dagger_{e3}\mathcal{K}_{3e} \sim 10^{-3}$ and $m_{E_1} \approx m_Y$ and $m_{E_2,E_3} \neq m_Y$, we obtain values for the three μ compatible with the values given in Eq. (1). We see from Fig. 2 that for a given j if $m_{E_j} \neq m_Y$ the respective F-factor contribution to Eq. (6) is of the order of one. However in Fig. 2 we do not include the $\sum_j \mathrm{Re}(K^\dagger_{ij}K_{ji})$ factor appearing in Eq. (6). The later factor is important for getting μ_ν compatible with the constraints given in Eq. (1).

For the case *ii)*, when the condition for the enhancement $m_\nu/v_s \approx 1$ is valid only for the heavier neutrino; μ-values compatible with those in Eq. (1) are also obtained but in this case there are suppression factors m_{ν_1}/m_{ν_3} and m_{ν_2}/m_{ν_3}.

Notice that the magnetic moments can be of the diagonal or transition type, hence for the ν_e case the phenomenological consideration of the resonant spin-flavor precession solution of the solar neutrino problem is valid [17]. So far all considerations are valid independently of the models. Any model which contains interactions like those in Eq. (3) will produce a magnetic moment with the characteristic discussed above. For example, in multi-Higgs extension of the standard model, say with several doublets with at least one of them coupling only to the leptons (by imposing an appropriate symmetry) plus a complex (non-majoron) triplet [18]. In this case it is possible to have FCNC in the charged lepton sector and the neutrino masses are $m_\nu = G^\nu v_s$, where v_s is the VEV of the neutral component of the triplet and G^ν is a complex symmetric 3×3 matrix. There are also models based on the $SU(3)_L \otimes U(1)_N$ electroweak symmetry with *a)* the leptons in triplets $\psi = (\nu_l, l^-, E_l^+)^T$ [19] or, *b)* $\psi = (\nu_l, l^-, l^+)^T$, $l = e, \mu, \tau$ [20]. In both models we have to add right-handed neutrinos. In the first model it is necessary also to add a scalar sextet S which is not needed in the minimal version of the model and we denote the VEV which give a contribution to the neutrino mass as v_1. (The other neutral component of the sextet can give contributions to the charged lepton masses.) In this situation, the Yukawa interactions are

$$-\mathcal{L}_l = \frac{G_{ab}}{\sqrt{2}}\,\overline{(\psi_{aiL})^c}\,\psi_{bjL}S_{ij} + \frac{1}{2}G'_{ab}\overline{(\psi_{aiL})^c}\,\nu_{bR}\eta + H.c., \tag{9}$$

and the mass matrix of the neutrinos are of the form

$$\begin{pmatrix} Gv_s & \frac{1}{2}G'v_\eta \\ \frac{1}{2}G'v_\eta & M^R \end{pmatrix} \tag{10}$$

where M^R is a possible Majorana mass term for the right-handed singlets which we have not included in Eq. (9). Hence, the biunitary matrices which diagonalize the mass matrix in Eq. (10) do not diagonalize separately neither Gv_S nor $G'v_\eta$, and there are flavor changing neutral interactions in the lepton-Yukawa sector. The interactions with the charged scalar are like those in Eq. (3) where E_j denote exotic charged leptons [19] in the *a)* case, or the

usual antileptons e^+, μ^+, τ^+ [20] in the *b)* one. Notice that now in Eq. (5) we have used $M^\nu/v_s \to G' \approx 1$. If the neutral interactions of the leptons E_j also violate flavor, both vertices in the diagram in Fig. 1 are not proportional to the lepton masses and the model is of the type of the model of Ref. [16]. The same happens with the first vertex (from the left) if the exotic leptons E_j have mixing with the known leptons. This is the case of the 331 model of Ref. [20]. In both models neutrinos are Majorana particles and may still have transition magnetic moments. However the general sort of models which are parameterized like in Eqs. (3) or (5), neutrinos can be Dirac particles with diagonal and transition dipole magnetic moments. In models with exotic charged leptons there is no contribution to the $\mu \to e\gamma$ decay if the exotic leptons do not couple to the usual known leptons as in Ref. [19]. In that model the contribution to the $\mu \to e\gamma$ decay arise via doubly charged scalars. On the other hand in models with only the known leptons the constraints coming from the decay $\mu \to e\gamma$ are [15]

$$\frac{\mathcal{K}^\dagger_{e2}\mathcal{K}_{2e}}{m_Y^2} < 10^{-8}\,\mathrm{GeV}^{-2}, \tag{11}$$

which constrains only the μ_{ν_μ}.

Of course, neutrinos have to have a mass different from zero for having $\mu_\nu \neq 0$ but, as we said before, that mass is arbitrary and could be rather small. We have been able to give a mechanism to generate a μ_ν which can be in the range 10^{-13}–$10^{-11}\mu_B$ *i.e.*, as large as the current upper limit coming from the supernovae collapse [21]

$$\mu_{\nu_e} < (0.1 - 0.4) \times 10^{-11}\mu_B, \tag{12}$$

or those in Eq. (1), even for a neutrino with an eV mass or smaller. However if their masses are small the respective mass square differences must also be very small and could be compatible with all experimental data coming from neutrino experiments like accelerator [22], solar [23], and atmospheric [24] ones.

Up to now the analyses of atmospheric neutrinos were restricted by events induced by the charged currents (CC) interactions (e-like and μ-like events). However, events induced by neutral currents (NC) can give important information on the neutrino flavor oscillations [25]. For example, a precise measurement of the ratio of the π^0-like events to the e-like events [26] can be used to distinguish $\nu \to \nu_\tau$ from $\nu_\mu \to \nu_s$ sterile neutrino oscillation. However, if there exist a large muon neutrino magnetic moment (diagonal or transition), it will produce an additional neutral current effect which has to be separated out to draw a definite conclusion [27]. We have seen that it is possible to have neutrinos with a magnetic dipole moment as large as $(10^{-11} - 10^{-10})\mu_B$ even with masses compatible with the mass square differences needed in LSND [22], solar [23] and atmospheric [24] neutrinos data.

Finally, notice that in Eq. (6) if K is a general unitary matrix, the interactions in Eq. (3) will induce electric dipole moments too.

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FIGURES

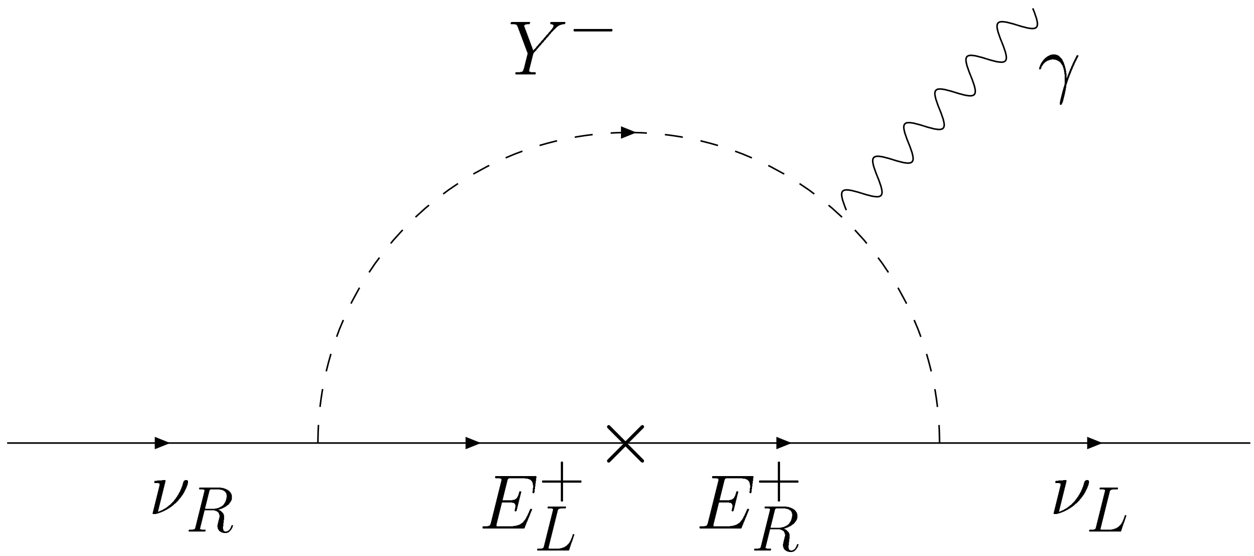

FIG. 1. One loop contributions to the magnetic moment of the neutrinos.

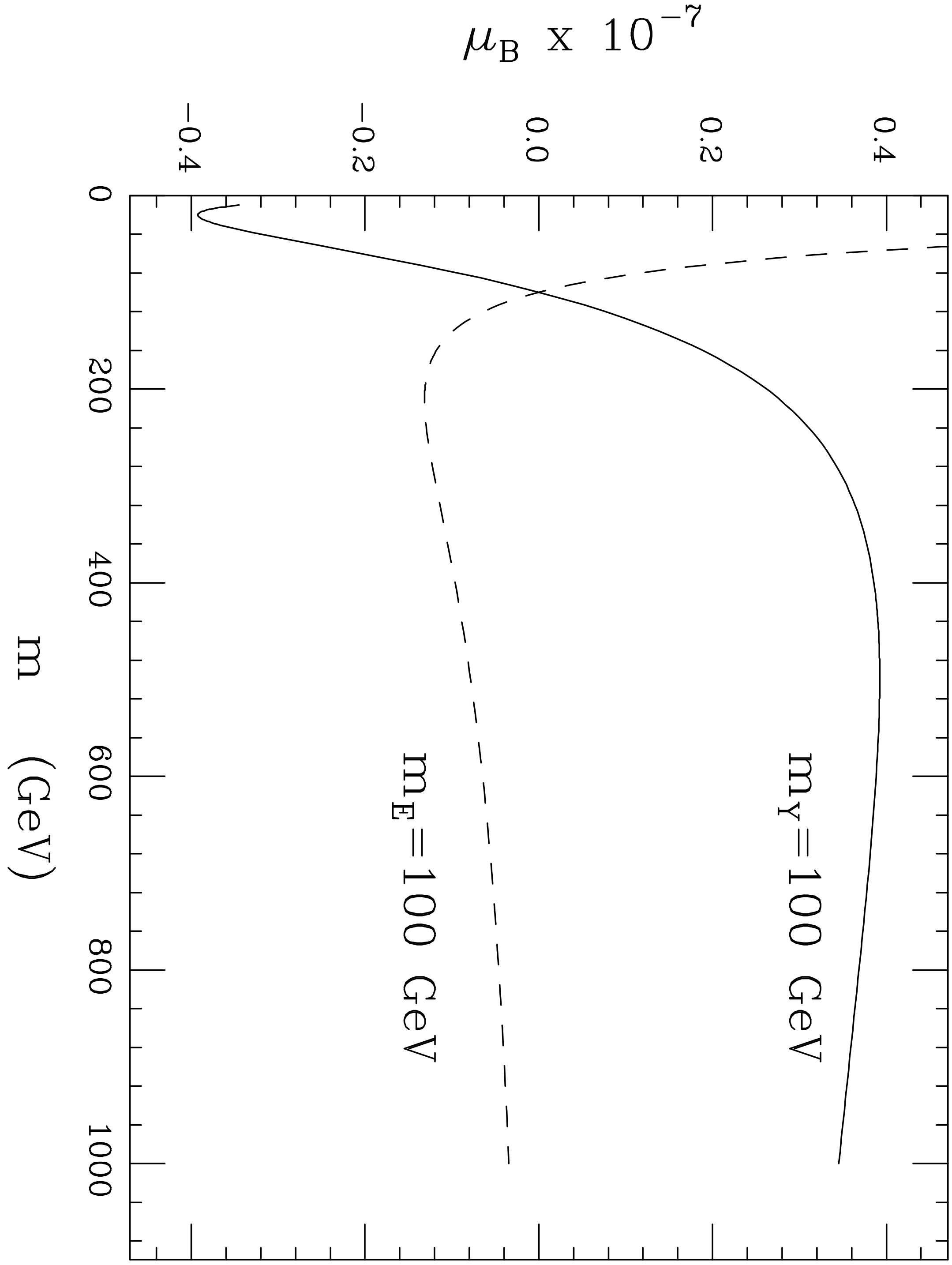

FIG. 2. The neutrino magnetic moment from Eq. (7) up to the mixing factor and for a fixed j and m_Y (m_E) as a function of m_E (m_Y).